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Surface roughness influence on parametric amplification of nanoresonators in presence of thermomechanical and environmental noise

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We investigate the surface roughness influence on the gain from parametric amplification in nanoresonators in the presence of thermomechanical and momentum exchange noise. The roughness is characterized by the rms amplitude w , the correlation length ξ , and the roughness exponent $0 < H < 1$. It is found that the gain strongly increases with increasing roughening (decreasing H and/or increasing ratio w/ξ) due to the increment in capacitive coupling, which plays a dominant role when the intrinsic quality factor Q_{in} is comparable or lower than the quality factor Q_{gas} due to gas collisions. However, for $Q_{\text{in}} \gg Q_{\text{gas}}$, the influence of surface roughness on the gain strongly diminishes. © 2008 American Institute of Physics. [DOI: 10.1063/1.2970108]

The detection of small forces and the mass of molecules adsorbed on surfaces are of interest in a wide area of research fields, such as scanning probe microscopy, gravity wave detection, and mass sensor technologies.^{1–3} Typically, the detection includes the conversion of mechanical motion to electrical signal via a transducer and then amplifying the electrical signal. A mechanical parametric amplifier greatly improves the mechanical response of micro/nanocantilevers responding to small harmonic forces.⁴ In this case, the possibility of squeezed thermomechanical noise has also been demonstrated.⁴ Recently it was also shown that in mass sensing at long averaging times, where the noise close to the carrier is sampled, the Allan variance (which gives the limit to mass sensitivity) becomes worse with increasing parametric amplification.⁵ Indeed, it was shown that the parametric amplification did not give improved performance over that achieved in the linear regime.⁵

Nonetheless, it remains unexplored how parametric amplification will be influenced by environmental noise due to the impingement of gas molecules,⁶ besides thermomechanical noise, if the resonator surfaces are rough. This source of noise is influenced by the surface morphology of the oscillating cantilever,⁷ which is a possibility to be further explored for parametric sensing. Notably, the influence of surfaces on nanoelectromechanical systems (NEMSs) has been shown in a variety of studies. Indeed, NEMSs of SiC/Si have been shown to be operational in the UHF/microwave regime when having low surface roughness, while devices with rougher surfaces could not be operated higher than the VHF regime.⁸ Studies of Si nanowires have shown the quality factor to decrease by increasing surface area to volume ratio.⁹ Recently random surface roughness was also shown to affect the quality factor, dynamic range, adsorption-desorption noise, and limit to mass sensitivity of nanoresonators.^{7,10} Therefore, these considerations motivate the present work to

explore how surface dependent fluctuation processes can influence parametric amplification.

We base our calculation on the simple harmonic oscillator model for the cantilever vibration $u(t)$ for its free end, having an effective oscillating mass M_{eff} and spring constant k_{eff} that result in a resonance frequency $\omega_o = (k_{\text{eff}}/M_{\text{eff}})^{1/2}$. For parametric sensing, a modulation of the spring constant $k(t)$ at a frequency of $2\omega_o$ is added, which is controlled using, e.g., a capacitive coupling between the cantilever on top of a fixed electrode.⁴ The advantage of parametric sensing is that as $k(t)$ is increased in amplitude, the response of the resonator to a weak external driving force $F(t)$ (representing the signal to be detected) is significantly amplified for drive frequencies near to ω_o .^{4,5}

The detected force is assumed to have the form $F(t) = F_o \cos(\omega_o t + \varphi)$ with φ as the phase angle between this external modulation and the independent actuating drive, and the pump voltage in the form of $\tilde{V}(t) = V_o + V \sin(2\omega_o t)$ (representing a particular degenerate case for this choice of frequencies). The latter yields a time varying spring constant $k(t) = k_c \sin(2\omega_o t)$ with $k_c = V_o V (\partial^2 C / \partial x^2)$ and $C(x)$ as the capacitance between the cantilever and ground electrode.⁴ The resonator's equation of motion has the form^{4,5}

$$M_{\text{eff}} \ddot{u} + [k_{\text{eff}} + k(t)]u = - (M_{\text{eff}} \omega_o / Q_{\text{in}}) \dot{u} - (M_{\text{eff}} \omega_o / Q_{\text{gas},r}) \dot{u} + F(t), \quad (1)$$

where $-(M_{\text{eff}} \omega_o / Q_{\text{in}}) \dot{u}$ is the force due to the intrinsic damping associated with thermomechanical noise, which is due to coupling between the resonator and its dissipative reservoir with intrinsic quality factor Q_{in} . The resonator can also undergo gas damping due to impingement and momentum exchange of gas molecules on its surface^{6,7} yielding a drag force $-(M_{\text{eff}} \omega_o / Q_{\text{gas},r}) \dot{u}$ with quality factor $Q_{\text{gas},r} = M_{\text{eff}} \omega_o \sqrt{K_B T / m (P A_{\text{rou}})^{-1}}$. P is the gas pressure, m is the molecule mass, and A_{rou} is the rough surface area of the beam.^{6,7} Here we assumed that the resonator operates within the molecular regime or molecule mean free path L_{mph} larger than the beam width w_b (\ll beam length L).¹¹ The gain is

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defined as the maximum displacement amplitude for $V > 0$ divided by that for $V = 0$ and given as (below the threshold for self-sustained oscillations)⁴

$$\text{Gain} = \{\cos^2 \varphi (1 + V/V_t)^{-2} + \sin^2 \varphi (1 - V/V_t)^{-2}\}^{1/2}, \quad (2)$$

where $V_t = 2k_{\text{eff}}/QV_o(\partial^2 C/\partial x^2)$. Q is the total quality factor where $1/Q = 1/Q_{\text{in}} + 1/Q_{\text{gas},r}$.

In order to further compute the gain, we have to compute Q and $Q_{\text{gas},r}$ due to gas dissipation. If we assume for the roughness profile of the resonator surface a single valued random function $h(r)$ of the in-plane position $r = (x, y)$ and a Gaussian height distribution,¹² the rough area is given by $A_{\text{rou}}/A_{\text{flat}} = R_{\text{rou}} = \int_0^{+\infty} du (\sqrt{1 + \rho^2 u}) e^{-u}$ (Ref. 13) with $\rho = \sqrt{\langle (\nabla h)^2 \rangle} = (\int_0^{+\infty} q^2 \langle |h(q)|^2 \rangle d^2 q)^{1/2}$ as the average local surface slope¹⁴ and $A_{\text{flat}} = w_b L$. $\langle |h(q)|^2 \rangle$ is the roughness spectrum and $Q_c = \pi/a_o$ with a_o as the lower lateral cutoff. If we substitute in Eq. (2) $C \sim R_{\text{rou}}$ (yielding $\partial^2 C/\partial x^2 \approx R_{\text{rou}} \partial^2 C_{\text{flat}}/\partial x^2$) by taking into account the fact that the capacitance $C(x)$ is proportional to surface area and thus to R_{rou} , one obtains for the gain the final form

$$\begin{aligned} \text{Gain} \approx & \{\cos^2 \varphi [1 + (V/V_{t,\text{in}} R_{\text{rou}}^{-1} (1 + Q_{\text{in}} R_{\text{rou}}/Q_{\text{gas},f}))]^{-2} \\ & + \sin^2 \varphi [1 - (V/V_{t,\text{in}} R_{\text{rou}}^{-1} (1 \\ & + Q_{\text{in}} R_{\text{rou}}/Q_{\text{gas},f}))]^{-2}\}^{1/2}, \end{aligned} \quad (3)$$

where $V_{t,\text{in}} = Q_{\text{in}}^{-1} [2k_{\text{eff}}/V_o(\partial^2 C_{\text{flat}}/\partial x^2)]$ and $Q_{\text{gas},f} = M_{\text{eff}} \omega_o \sqrt{K_B T/m(PA_{\text{flat}})^{-1}}$.

The gain calculations in terms of Eq. (3) were performed for random self-affine rough surfaces, where it is observed in a wide spectrum of surface engineering processes.¹² In this case $\langle |h(q)|^2 \rangle$ scales as $\langle |h(q)|^2 \rangle \propto q^{-2-2H}$ if $q\xi \gg 1$, and $\langle |h(q)|^2 \rangle \propto \text{const}$ if $q\xi \ll 1$.^{12,15} This is satisfied by the analytic model¹⁵ $\langle |h(q)|^2 \rangle = (2\pi w^2 \xi^2)/(1 + a q^2 \xi^2)^{(1+H)}$ with $a = (1/2H) \times [1 - (1 + a Q_c^2 \xi^2)^{-H}]$ if $0 < H < 1$, and $a = 1/2 \ln(1 + a Q_c^2 \xi^2)$ if $H = 0$. Small values of H (~ 0) characterize jagged or irregular surfaces, while large values of H (~ 1) surface with smooth hills-valleys.^{12,15} In addition, we obtain for the local slope the analytic expression $\rho = (w/\sqrt{2}\xi a)\{(1-H)^{-1}[(1 + a Q_c^2 \xi^2)^{1-H} - 1] - 2a\}^{1/2}$,¹⁴ which further facilitates calculations of the gain from Eq. (3). For other roughness models see Ref. 16.

The numerical calculations were performed for roughness amplitudes observed in real nanoresonator surfaces $w \sim 3-10$ nm,⁸ and for the cutoff we used $a_o = 0.3$ nm. Figures 1 and 2 show calculations of the gain for two different phase angles φ close to the deamplifying regime $\varphi \sim 0$ up to full amplification that occurs for $\varphi = 90^\circ$. Even for small φ , some amplification can occur for pump voltages V close to the threshold value V_t due to contribution from the amplifying term (proportional to $\sin^2 \varphi$) in Eq. (3). Moreover, as Fig. 3 indicates in comparison to Fig. 1, with decreasing gas quality factor $Q_{\text{gas},f}$ (so that $Q_{\text{gas},f} < Q_{\text{in}}$) the amplification occurs at larger pumping voltages V since in this case V_t increases significantly. Therefore, the gas environment plays a significant role in parametric amplification.

Furthermore, Figs. 1–3 show calculations of the gain for various roughness exponents H . If we compare Figs. 1 and 2, it becomes evident that the influence of surface roughening

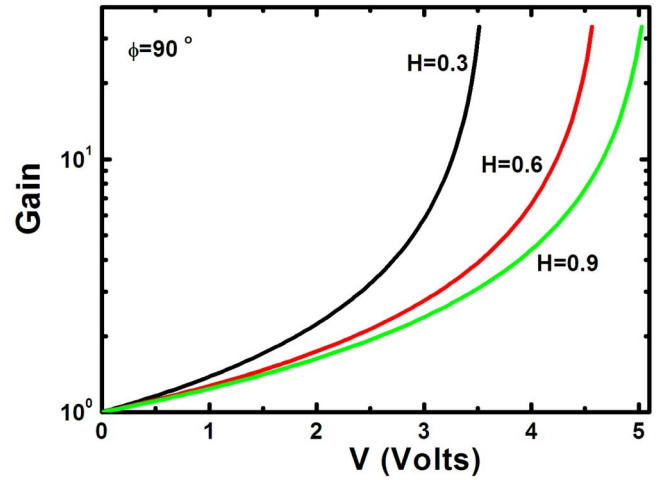


FIG. 1. (Color online) Gain vs pumping voltage V for phase angles $\varphi = 90^\circ$, H as indicated, $\xi = 60$ nm, $w = 3$ nm, $Q_{\text{in}}/Q_{\text{gas},f} = 1$, and $Q_{\text{in}} = 10^4$.

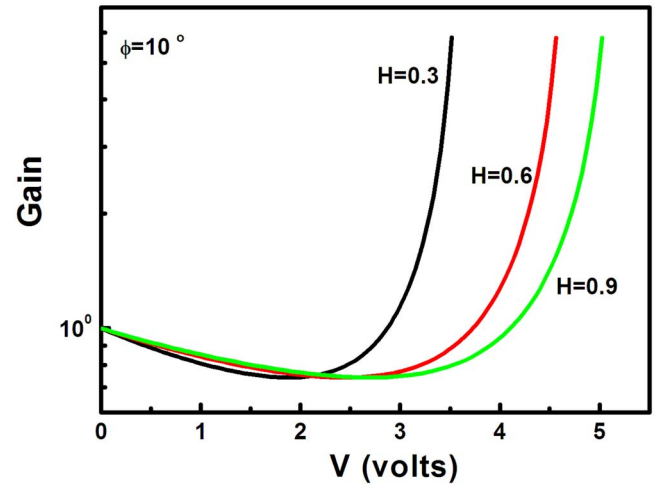


FIG. 2. (Color online) Gain vs pumping voltage V for phase angles $\varphi = 10^\circ$, H as indicated, $\xi = 60$ nm, $w = 3$ nm, $Q_{\text{in}}/Q_{\text{gas},f} = 1$, and $Q_{\text{in}} = 10^4$.

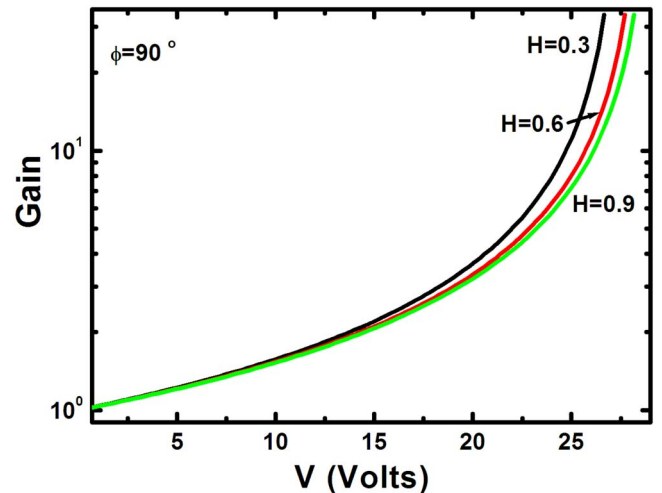


FIG. 3. (Color online) Gain vs pumping voltage V for phase angles $\varphi = 90^\circ$, H as indicated, $\xi = 60$ nm, $w = 3$ nm, $Q_{\text{in}}/Q_{\text{gas},f} = 10$, and $Q_{\text{in}} = 10^4$.

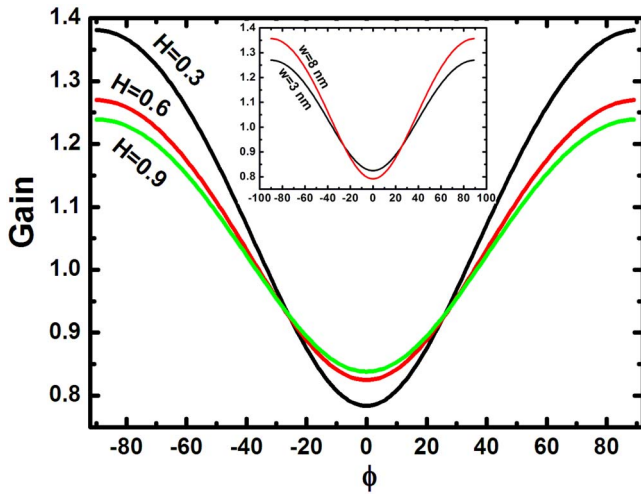


FIG. 4. (Color online) Gain vs phase angle ϕ , pumping voltage $V=1$ V, H as indicated, $\xi=60$ nm, $w=3$ nm, $Q_{in}/Q_{gas,f}=1$, and $Q_{in}=10^4$. The inset shows similar calculations for $H=0.6$ and different amplitudes w as indicated.

becomes more distinct for pump voltages V close to V_t . Indeed, decreasing H (equivalently increasing roughness at short length scales $<\xi$) leads to decrement in the critical voltage V_t where maximum amplification occurs. Comparing Figs. 1 and 3, one can conclude that the gain increases with increasing roughening (i. e., decreasing H and/or increasing ratio w/ξ) due to increment in the capacitive coupling associated to increasing surface area, which however plays a dominant role when the intrinsic quality factor is comparable or lower than that due to gas collisions. In the opposite case $Q_{in} \gg Q_{gas,f}$, the influence of surface roughness on the gain strongly diminishes because the increment in gas dissipation and the increment in capacitive coupling counterbalance each other. Indeed, from Eq. (3) we obtain for $Q_{in} \gg Q_{gas,f}$

$$\text{Gain} \approx \{\cos^2 \phi [1 + (V/V_{t,in})(Q_{gas,f}/Q_{in})]^{-2} + \sin^2 \phi [1 - (V/V_{t,in})(Q_{gas,f}/Q_{in})]^{-2}\}^{1/2}, \quad (4)$$

which indicates that the gain does not depend on surface roughness.

Finally, if we compare Figs. 1 and 2, it is evident that for large phase angles ϕ (close to the maximum amplifying regime of $\sim 90^\circ$), increasing roughness leads to higher amplification, while for small angles ϕ (close to the deamplifying value $\phi=0$) the influence of surface morphology is not significant for $V \ll V_t$. For clarity, Fig. 4 shows the direct plot of the Gain versus phase angle ϕ for various exponents H and a

sufficiently large pumping voltage V (comparable to V_t). As shown in Fig. 4, increasing roughness (i. e., decreasing H in these schematics) leads to higher amplification for phase angles $\phi \sim 90^\circ$, while for phase values close to $\phi \sim 0^\circ$ it leads to higher deamplification. Similar is the effect of increasing roughness amplitude w and/or decreasing correlation length ξ (see inset in Fig. 4) since in both cases surface roughening occurs.

In conclusion, we investigated the simultaneous influence of thermomechanical and momentum exchange noise on the gain of nonlinear parametric amplification. It is found that the amplification gain strongly increases with increasing roughening due to increment in capacitive coupling, which plays a dominant role when the intrinsic quality factor is comparable or lower than the quality factor due to gas collisions. This result will hold qualitatively also for non-self-affine roughness models as long as variations in the characteristic roughness parameters lead to rougher surfaces. In the opposite case, the influence of surface roughness is negligible. Notably, these considerations should be taken into account in real parametric amplifiers with nanoscale rough surfaces.

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